

# The Origin of the Mass-Metallicity relation: an analytical approach

E. Spitoni<sup>1</sup> \*, F. Calura<sup>2</sup>, F. Matteucci<sup>1,2</sup>, and S. Recchi<sup>3</sup>

<sup>1</sup> Dipartimento di Fisica, Sezione di Astronomia, Università di Trieste, via G.B. Tiepolo 11, I-34131

<sup>2</sup> I.N.A.F. Osservatorio Astronomico di Trieste, via G.B. Tiepolo 11, I-34131

<sup>3</sup> Institute of Astronomy, Vienna University, Türkenschanzstrasse 17, A-1180, Vienna, Austria

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## ABSTRACT

**Context.** The existence of a mass-metallicity (MZ) relation in star forming galaxies at all redshift has been recently established. It is therefore important to understand the physical mechanisms underlying such a relation.

**Aims.** We aim at studying some possible physical mechanisms contributing to the MZ relation by adopting analytical solutions of chemical evolution models including infall and outflow.

**Methods.** Analytical models assume the instantaneous recycling approximation which is still an acceptable assumption for elements produced on short timescales such as oxygen, which is the measured abundance in the MZ relation. We explore the hypotheses of a variable galactic wind rate, infall rate and yield per stellar generation (i.e. a variation in the IMF), as possible causes for the MZ relation.

**Results.** By means of analytical models we compute the expected O abundance for galaxies of a given total baryonic mass and gas mass. The stellar mass is derived observationally and the gas mass is derived by inverting the Kennicutt law of star formation, once the star formation rate is known. Then we test how the parameters describing the outflow, infall and IMF should vary to reproduce the MZ relation, and we exclude the cases where such a variation leads to unrealistic situations.

**Conclusions.** We find that a galactic wind rate increasing with decreasing galactic mass or a variable IMF are both viable solutions for the MZ relation. A variable infall rate instead is not acceptable. It is difficult to disentangle among the outflow and IMF solutions only by considering the MZ relation, and other observational constraints should be taken into account to select a specific solution. For example, a variable efficiency of star formation increasing with galactic mass can also reproduce the MZ relation and explain the downsizing in star formation suggested for ellipticals. The best solution could be a variable efficiency of star formation coupled with galactic winds, which are indeed observed in low mass galaxies.

**Key words.** ISM: abundances - Galaxies: abundances - Galaxies: stellar content - Galaxies: evolution.

## 1. Introduction

The study of the relation between the galactic stellar mass and the gas-phase metallicity, which represents the abundance of heavy elements present in the interstellar medium (hereinafter ISM), provides us with constraints on various parameters fundamental for any galaxy formation theory. These parameters include the stellar initial mass function (IMF), representing the distribution of stellar masses at birth, which strongly influences the amount of heavy elements produced and restored by stars into the ISM. The mass-metallicity (hereinafter MZ) relation can also provide information on the star formation history: in general, a higher star formation rate produces a larger concentration of metals in the ISM. Finally, the MZ relation provides with information on the role of infall/outflow in galaxy evolution, namely on the amount of matter either ejected from galaxies into the intergalactic medium (IGM) or accreted onto galaxies from the IGM. Galactic outflows are directly con-

nected to the importance of feedback and energy exchange between stars and interstellar gas. At the present time, different theoretical explanations of the MZ relation have been proposed by various authors. The first explanation is based on starburst-induced galactic outflows, more efficient in expelling metal-enriched matter from low-mass galaxies than from giant galaxies, mainly owing to the shallower gravitational potential wells of the former (Larson 1974; Dekel & Silk 1986; Tremonti et al. 2004; De Lucia et al. 2004; Kobayashi et al. 2007; Finlator & Davé 2008). However, at the present time it is difficult to assess how the outflow efficiency depends on the baryonic galactic mass. In an alternative scenario, infall of pristine gas can dilute the interstellar metals and act in the same way as outflows, once one assumes longer infall time scales in lower mass galaxies (Dalcanton et al. 2004, but see also Dalcanton 2007). Another explanation is that dwarf galaxies are less evolved than large galaxies, namely that the efficiency of star formation is larger in more massive systems. In this picture, large galaxies have formed the bulk of their stars by means of an intense star formation event at high redshift, quickly enriching their ISM to so-

\* email to: spitoni@oats.inaf.it

lar or over-solar metallicities, whereas dwarf galaxies, characterized by lower star formation efficiencies (i.e. star formation rates per unit mass of gas) have sub-solar interstellar metallicities. This interpretation is supported by various chemical evolution studies (Lequeux et al. 1979; Matteucci 1994; Calura et al. 2009), by cosmological N-body simulations (Brooks et al. 2007; Mouhcine et al. 2008; Tassis et al. 2008) and by hydrodynamical simulations (Tissera et al. 2005; De Rossi et al. 2007). A fourth way to produce a MZ relation is by means of a variable IMF. Köppen et al. (2007) showed how a higher upper mass cutoff and a flatter slope in the IMF in more massive galaxies can account for the observed MZ relation. It is possible that the relative importance of these processes may vary as a function of the total galactic stellar mass. However, the theoretical investigations performed so far do not allow us to disentangle among these phenomena and to assess which may be considered dominant in galaxies of different mass.

In this paper, we aim at investigating the importance of different physical processes in determining the MZ relation, by means of analytical chemical evolution models. This analytical approach will allow us to rapidly test the parameter space involved in our study, and it is justified by the fact that the oxygen evolution can be computed under the instantaneous recycling approximation, required to obtain analytical solutions. By means of the solutions of the analytical models including infall and outflow we can compute the expected metallicity for a galaxy of a given mass and a given gas mass fraction. The total stellar mass and gas mass are derived from the observational data: in particular, we use the local MZ relation determined in SDSS galaxies, and from the SFRs observed in the same galaxy sample, we derive the gas mass fractions by adopting the Kennicutt (1998) law for star formation. We aim at assessing the importance of outflow, infall and IMF (through the yield per stellar generation) in determining the MZ relation for galaxies of various masses. We also aim at providing detailed expressions for the effective yields in different physical conditions, that can be used to compare with the empirical effective yields as derived from observations (see Dalcanton 2007).

Our paper is organized as follows. In section 2, we present our equations, our basic assumptions and the observables used in our study. In Section 3, we present our results and finally, in Section 4, some conclusions are drawn.

## 2. Summary of simple model solutions

### 2.1. The Simple model

We recall here the main assumptions and results of the simple models of chemical evolution (Tinsley 1980; Matteucci 2001; Recchi et al. 2008). As it is well known, the so-called *Simple Model* of chemical evolution is based on the following assumptions:

1. the system is one-zone and closed, namely there are no inflows or outflows.
2. The initial gas is primordial (no metals).

3. The IMF is constant in time.
4. The gas is well mixed at any time (*instantaneous mixing approximation*).
5. Stars more massive than  $1 M_{\odot}$  die instantaneously; stars smaller than  $1 M_{\odot}$  live forever (*instantaneous recycling approximation* or IRA).

These simplifying assumptions allow us to calculate analytically the chemical evolution of the galaxies. Once we have defined the fundamental quantities, such as the returned fraction:

$$R = \int_1^{\infty} (m - M_R) \phi(m) dm, \quad (1)$$

(where  $\phi(m)$  is the IMF and  $M_R$  is the mass of the remnant) and the yield per stellar generation:

$$y_Z = \frac{1}{1 - R} \int_1^{\infty} m p_{Z,m} \phi(m) dm, \quad (2)$$

(where  $p_{Z,m}$  is the fraction of newly produced and ejected metals by a star of mass  $m$ )

The well known solution of the closed box model can be easily found:

$$Z = y_Z \ln(\mu^{-1}) \quad (3)$$

where  $\mu$  is the gas fraction  $M_{\text{gas}}/M_{\text{tot}}$ , with  $M_{\text{tot}} = M_{\star} + M_{\text{gas}}$ . This result is obtained by assuming that the galaxy initially contains only gas and has the remarkable property that it does not depend on the particular star formation history the galaxy experiences. Moreover, to obtain eq. (3) one has to assume that  $y_Z$  is constant in time. The yield which appears in eq. (3) is known as effective yield,  $y_{Z,\text{eff}}$ , namely the yield the system would have if described by the simple model. In models with gas flows the true yield, as defined in eq. (2), does not coincide with the effective yield, as we will see in the next paragraphs.

### 2.2. Leaky box models

Relaxing the first of the assumptions of the simple model, we get the models including gas flows, also known as *leaky box* models. Analytical solutions of simple models of chemical evolution including infall or outflow are known since at least 30 years (Pagel & Patchett 1975 ; Hartwick 1976 ; Twarog 1980; Edmunds 1990 ). Here we follow the approach and the terminology of Matteucci (2001), namely we assume for simplicity *linear* flows (we assume gas flows proportional to the star formation rate (SFR)). Therefore, the outflow rate  $W(t)$  is defined as:

$$W(t) = \lambda(1 - R)\psi(t), \quad (4)$$

where  $\psi(t)$  is the SFR, and the infall rate  $A(t)$  is given by:

$$A(t) = \Lambda(1 - R)\psi(t). \quad (5)$$

Here  $\lambda$  and  $\Lambda$  are two proportionality constants  $\geq 0$ . The first assumption is justified by the fact that, the larger the SFR is, the more intense are the energetic events associated with it (in

particular supernova explosions and stellar winds) and therefore the larger is the chance of having a large-scale outflow (see e.g. Silk 2003). A proportionality between  $A(t)$  and  $\psi(t)$  is less easily justifiable physically.

If we consider a system including outflow and infall the set of equations we need to solve is the following one:

$$\begin{cases} \frac{dM_{tot}}{dt} = (\Lambda - \lambda)(1 - R)\psi(t) \\ \frac{dM_{gas}}{dt} = (\Lambda - \lambda - 1)(1 - R)\psi(t) \\ \frac{dM_Z}{dt} = (1 - R)\psi(t)[\Lambda Z_A + y_Z - (\lambda + 1)Z] \end{cases} \quad (6)$$

where  $M_Z$  is the mass of metals ( $M_Z = Z \cdot M_{gas}$ ) and  $Z_A$  is the metallicity of the infalling gas.

### 2.2.1. Simple Model with outflow

If we study a simple model with only outflow, e.g.  $A(t) = 0$ , the system of equation (6) can be worked out, yielding to the following solution (Matteucci 2001):

$$Z = \frac{y_Z}{(1 + \lambda)} \ln[(1 + \lambda)\mu^{-1} - \lambda]. \quad (7)$$

For the integration we assume that at  $t = 0$ ,  $Z(0) = 0$ ,  $M_{tot}(0) = M_{gas}(0) = M_{g,0}$ . Since  $\mu$  values span in the range between  $0 \leq \mu \leq 1$ , the solution (7) is well defined for all the values of  $\lambda \geq 0$ . In this case, the true yield is given by:

$$y_Z = \frac{Z(1 + \lambda)}{\ln[(1 + \lambda)\mu^{-1} - \lambda]} \quad (8)$$

and it is clearly lower than the *effective yield* defined in eq. (3). This fact should be kept in mind when using eq. (3) to interpret observational data.

### 2.2.2. Simple Model with infall

Here we analyze the opposite case:  $A(t) \neq 0$  and  $W(t) = 0$ . Assuming for the infalling gas a primordial composition (e.g.  $Z_A = 0$ ),  $\Lambda \neq 1$  and  $\Lambda \neq 0$ , we obtain this solution (Matteucci 2001):

$$Z = \frac{y_Z}{\Lambda} \left\{ 1 - [(\Lambda - (\Lambda - 1)\mu^{-1})^{\frac{\Lambda}{\Lambda-1}}] \right\}. \quad (9)$$

We notice here that, in models with infall, not all the values of  $\mu$  are allowed. If  $\Lambda > 1$ ,  $\mu$  ranges between 1 and a minimum value:

$$\mu_{min} = \frac{\Lambda - 1}{\Lambda}. \quad (10)$$

In the case with  $\Lambda < 1$  there is no  $\mu_{min}$ . If  $\Lambda = 1$  the solution is:  $Z = y_Z[1 - e^{-(\mu^{-1}-1)}]$ , which is well known solution of the extreme infall (Larson 1972), where the amount of gas remains constant in time.

### 2.2.3. Simple Model with inflow and outflow

The general solution for a system described by the simple model in the presence of inflow of gas with a general metallicity  $Z_A$  and outflow is (Recchi et al. 2008):

$$Z = \frac{\Lambda Z_A + y_Z}{\Lambda} \left\{ 1 - [(\Lambda - \lambda) - (\Lambda - \lambda - 1)\mu^{-1}]^{\frac{\Lambda}{\Lambda-1}} \right\} \quad (11)$$

If the outflow rate is larger than the inflow rate the solution is defined for all  $\mu$  values, in the opposite case not all values of  $\mu$  are allowed.

In the models in which  $\Lambda > \lambda + 1$ ,  $M_{gas}$  is always increasing (eq. 6), therefore  $\mu$  ranges between 1 and a minimum value:

$$\mu_{min} = \frac{\Lambda - \lambda - 1}{\Lambda - \lambda}. \quad (12)$$

For models in which  $\lambda + 1 > \Lambda > \lambda$  there is no  $\mu_{min}$  but there is an upper limit reachable by the gas mass which is given by  $M_{gas,lim} = M_{g,0}/(\lambda + 1 - \Lambda)$ .

### 2.2.4. Simple Model with differential winds

Recchi et al. (2008) presented a set of new solutions in the simple model context in presence of differential winds, namely models where the metals are more easily channelled out of the parent galaxy than the pristine gas. The easiest way to consider a differential wind in the framework of simple models of chemical evolution is to assume that the metallicity of the gas carried out in the galactic wind is proportional to the metallicity of the ISM with a proportionality constant larger than one. If we define  $Z^o$  as the metallicity of the outflowing gas, this condition implies that  $Z^o = \beta Z$  with the ejection efficiency  $\beta > 1$ . In the metallicity budget (third equation in (6)) we assume that the negative term due to the galactic wind is given by  $W(t)Z^o = \beta\lambda(1 - R)\psi(t)$ .

With their simple approach, Recchi et al. (2008) were able to determine analytical expressions for the evolution of  $Z$ , which allow us to understand more clearly the effect of galactic winds on the chemical evolution of galaxies.

The set of the equations we solve in this case is very similar to (6), with the only difference given by the metallicity budget equation, which we modify as follows:

$$\frac{dM_Z}{dt} = (1 - R)\psi(t)[\Lambda Z_A + y_Z - (\lambda\beta + 1)Z]. \quad (13)$$

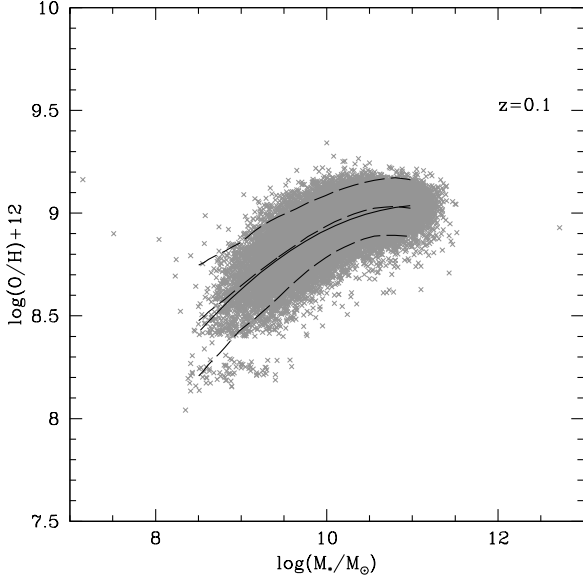
The solution of this new set of equations is given by:

$$Z = \frac{\Lambda Z_A + y_Z}{\Lambda + (\beta - 1)\lambda} \left\{ 1 - [(\Lambda - \lambda) - (\Lambda - \lambda - 1)\mu^{-1}]^{\frac{\Lambda + (\beta - 1)\lambda}{\Lambda - \lambda - 1}} \right\}. \quad (14)$$

It is trivial to see that we can obtain eq. (11) in the case  $\beta = 1$  (i.e. in the case in which the galactic wind is not differential).

## 3. The Observed Mass-metallicity relation and our method

Kewley & Ellison (2008) analyzed the metallicity relation for Sloan Digit Sky Survey (SDSS) 27,730 star-forming galaxies



**Fig. 1.** The observed MZ relation for the oxygen (Kewley & Ellison, 2008). The dashed lines are the average values and the standard deviation and the solid one is the best fit to the average values as given by Maiolino et al. (2008). The redshift of the galaxies is indicated at the top right of the figure.

by adopting 10 different metallicity calibrations. In Fig. 1 we show the MZ relation at  $z = 0.1$  obtained by using the calibration of Kewley & Dopita (2002), considered the best in the analysis by Calura et al. (2009), including the average values and the standard deviation indicated with the dashed lines.

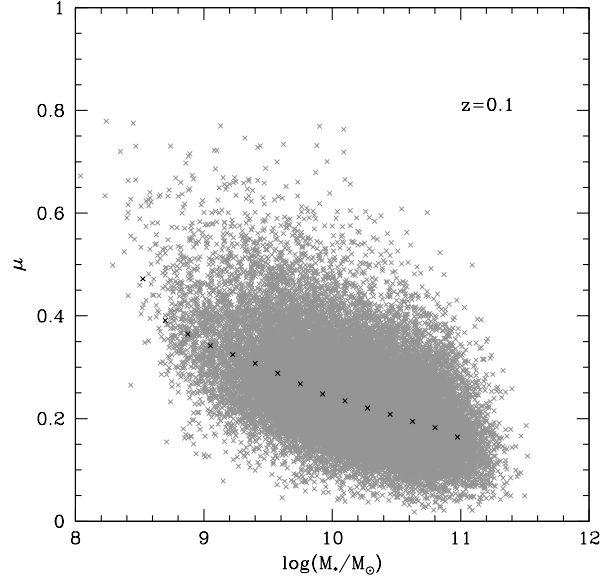
In the same figure the solid line is the analytical fit of the observed MZ relation of Kewley & Ellison (2008), as given by Maiolino et al. (2008), in particular:

$$\log(O/H) + 12 = -0.0864 * \left[ \log\left(\frac{M_*}{M_\odot}\right) - 11.18 \right]^2 + 9.04. \quad (15)$$

Since now on we will use this analytical fit as our fiducial MZ relation. For estimating the amount of gas which resides in each star-forming galaxy we use the method described in Calura et al. (2008). We aim at determining the cold gas mass of each galaxy on the basis of its SFR, by inverting the Kennicutt (1998) (hereafter K98) relation, which links the gas surface density to the SFR per unit area. A similar technique was used by Erb et al. (2006) and Erb (2008) to derive the gas fractions for a sample of star forming galaxies at  $z \sim 2$  and to study the implications of the MZ relation observed at high redshift on the galactic gas accretion history, respectively.

Following K98, for any galaxy, the gas surface density  $\Sigma_{gas}$ , expressed in  $M_\odot pc^{-2}$ , depends on the SFR surface density  $\dot{\Sigma}_*$ , expressed in  $M_\odot yr^{-1} kpc^{-2}$  according to:

$$\Sigma_{gas} = \left( \frac{\dot{\Sigma}_*}{2.5 \times 10^{-4}} \right)^{0.714} M_\odot pc^{-2} \quad (16)$$



**Fig. 2.** The gas fraction  $\mu$  as a function of the stellar galactic mass  $M_*$ . We divide the stellar masses in bins and the average  $\mu$  values for each bin are indicated by black crosses.

The scaling radius  $R_d$ , which will be used to compute the SFR surface density profile of each galaxy, can be calculated as (Mo et al. 1998):

$$R_d = \frac{\lambda_s R_{200} f_{c_{vir}}^{-1/2} f_R(\lambda_s, c_{vir}, f_b)}{\sqrt{2}}. \quad (17)$$

$\lambda_s$  is the spin parameter of the halo and depends on the total energy of the halo  $E$ , its angular momentum  $J$  and its mass  $M$  according to:

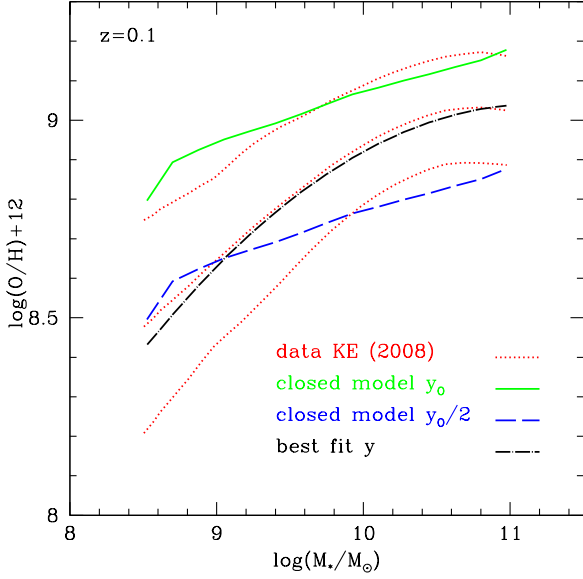
$$\lambda_s = J|E|^{1/2} G^{-1} M^{-5/2}. \quad (18)$$

The quantity  $\lambda_s$  is likely to assume values in the range  $0.01 \leq \lambda \leq 0.1$  (Heavens & Peacock (1988), Barnes & Efstathiou (1987), Jimenez et al. 1998). In this paper, we assume a typical value of  $\lambda_s = 0.05$ . Scatter in  $\lambda_s$  would propagate as scatter in the gas fractions, but we average over many galaxies to obtain the average gas fractions.

The parameter  $c_{vir}$  is the halo concentration factor, and is calculated following Bullock et al. (2001) and Somerville et al. (2006) i.e., by defining for each halo a collapse redshift  $z_c$ , as  $M_*(z_c) = F \cdot M$ .  $c_{vir}$  is given by  $c_{vir}(M, z) = K(1 + z_c)/(1 + z)$ , where  $F$  and  $K$  are two adjustable parameters. Following Somerville et al. (2006), we assume  $K = 3.4$  and  $F = 0.01$ . Here we assume  $z \sim 0.1$ , corresponding to the average redshift of the SDSS galaxies (Tremonti et al. 2004).

To compute the quantities  $f_{c_{vir}}$  and  $f_R$ , it was used an analytic fitting functions presented in Mo et al. (1998). For each galaxy, if  $\psi$  is the SFR in units of  $M_\odot/yr$  and if we assume for the SFR surface density profile  $\dot{\Sigma}_*(R) = \dot{\Sigma}_{*,0} \exp(-1.4R/R_d)$ , the central SFR surface density is then given by:

$$\dot{\Sigma}_{*,0} = \frac{\psi}{2\pi(R_d/1.4)^2}. \quad (19)$$



**Fig. 3.** The observed MZ relation and related standard deviation are indicated with the dotted red lines. With the green solid line we show our closed box model results assuming for the oxygen a yield per stellar generation of  $y_O = 0.01$ , whereas with the dashed blue line we show the model with  $y_O = 0.005$ . The model which best fits the data by using a variable yield is labelled with the short dashed dotted line.

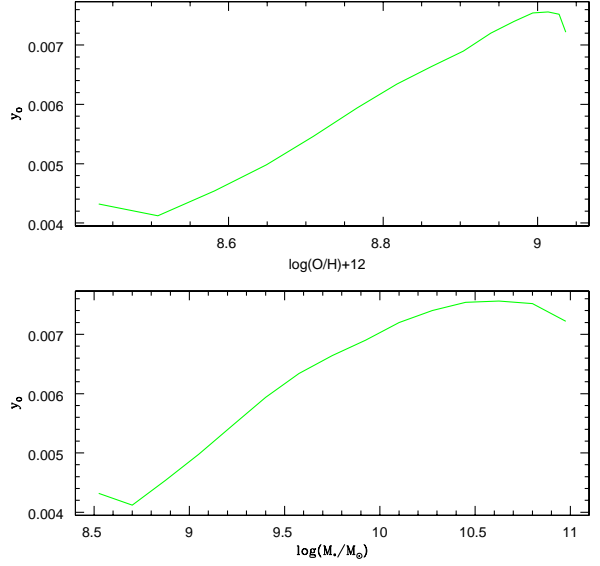
The gas surface density  $\Sigma_{gas}$  is then determined by the K98 relation, and the gas mass  $M_{gas}$  (in  $M_\odot$ ) is given by:

$$M_{gas} = \Sigma_{gas} \times 2\pi R_d^2. \quad (20)$$

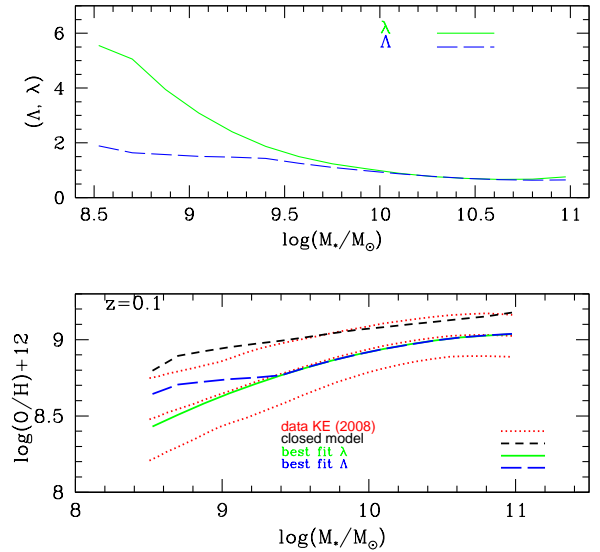
At this point we have a relation between  $\mu$  and the stellar mass for each considered galaxy as shown in Fig. 2.

The aim of this work is to constrain the model parameters of simple models to reproduce the analytical fit (eq. 15) of the MZ relation. In our reference model we assume for the oxygen yield the value of  $y_O = 0.01$ , this yield is obtained by adopting the Salpeter IMF ( $x=1.35$  over the mass range  $0.1-100M_\odot$ ), the same used for the inversion of the K98 law, and the stellar yields of Woosley & Weaver (1995) for oxygen at solar metallicity. In the first part of our work we explore the effect of a different choice of yields in the closed models. Given a stellar mass content  $M_*$ , we have an oxygen abundance value from the observed MZ relation, and then for each galaxy we have an estimate of the gas fraction  $\mu$  using the procedure described above. Therefore in closed models the only free parameter is the yield. We will show also the effect of a non-constant yield, assuming that is the IMF which varies and not the nucleosynthesis from galaxy to galaxy.

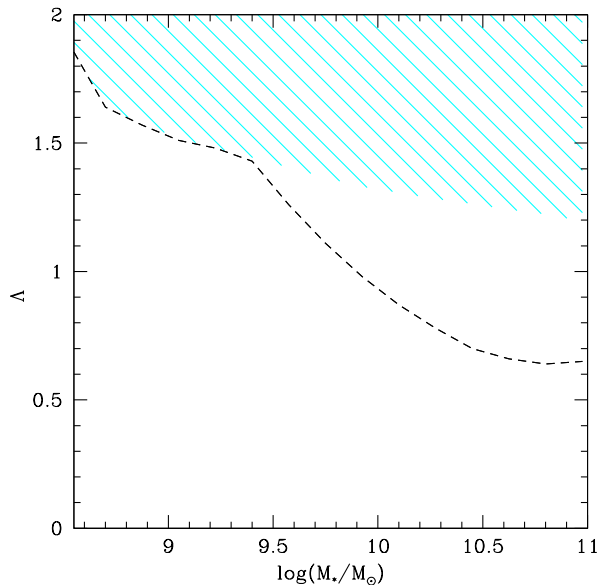
Concerning leaky box models if we fix  $Z$  and  $\mu$  we can vary the wind and infall parameters as well as the yield. We aim at testing the way in which the parameter space is constrained if we want to reproduce the observed MZ relation using simple model solutions.



**Fig. 4.** In the lower panel we report the  $y_O$  yield values as functions of the galactic stellar mass  $M_*$ , those obtained to reproduce the MZ relation, while in the upper panel we show the same yields as functions of the derived O abundances.



**Fig. 5.** *Upper panel:* The best  $\Lambda$  and  $\lambda$  values obtained for models with only infall and outflow, respectively, in order to reproduce the MZ relation.  $\Lambda$  and  $\lambda$  are given as functions of the galactic stellar mass  $M_*$ . *Lower panel:* The best fit to the observed MZ relation and related standard deviation are indicated with the dotted red lines. With the green solid line we show our best model results considering only outflow, while with the long dashed blue line the best results for the model with only infall are. With the short dashed line are indicated the results of the closed box model. In all models a value of  $y_O = 0.01$  is adopted.



**Fig. 6.** Case with only infall: dashed area represents the forbidden  $\Lambda$  values, the rest of the area represents the allowed values. With the short dashed line we draw the values of  $\Lambda$  as a function of  $\log(M_*/M_\odot)$  adopted in the MZ relation of Fig. 5.

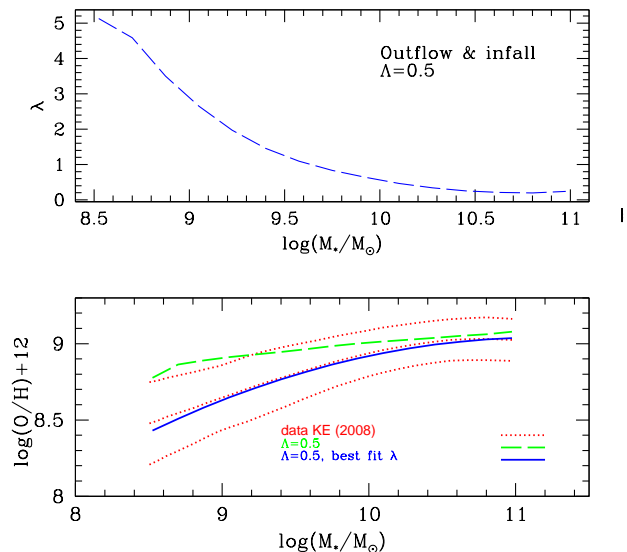
## 4. Results

### 4.1. Closed Box Model Results

First of all we report our results for the closed model. In this case we can vary only the yield in the eq. (3), because once fixed  $M_*$ , the  $Z$  is given by observational fit of Maiolino et al. (2008) and  $\mu$  from the inversion of the Kennicutt law. In the Fig (3) we report the simple model solution compared to the observational average values and the standard deviation for the MZ relation. If we consider the simple model adopting a constant oxygen effective yield of  $y_O=0.01$ , observational data are clearly not reproduced. The predicted slope is too flat as shown in Fig (3) with the green solid line. Even if we consider a smaller yield, the problem is not solved: the MZ is simply translated at lower values with the same slope (the blue dashed line in Fig 3 is the model with  $y_O=0.005$ ). The next step is to consider a variable yield as a function of the  $M_*$ . In the upper panel of Fig. 4 we report the yields per stellar generation as a function of the metallicity obtained to reproduce the MZ relation, whereas in the lower panel  $y$  is as a function of stellar mass. The best fit yields spans in the range between 0.004 and 0.007.

### 4.2. Leaky box Results

In this section we show our results in the Leaky box framework. We keep the oxygen yield fixed at  $y_O = 0.01$ . First of all we present results for systems in which only outflows (eq. 7) and only infall (eq. 9) are included, separately. In the upper panel of Fig 5 we report the best fit  $\Lambda$  and  $\lambda$  parameters as a function of stellar mass in galaxies for reproducing the MZ relation. Both the parameters, as expected, anti-correlate with the stellar mass



**Fig. 7.** *Upper panel:* the best  $\lambda$  values to reproduce the MZ relation in the framework of the model with both infall and outflow. The infall parameter is fixed to be  $\Lambda=0.5$ . *Lower panel:* the best fit to the observed MZ relation and related standard deviation are indicated with the dotted red lines. With the blue solid line we show the results of the best model with both outflow and infall, obtained by fixing  $\Lambda=0.5$  and varying  $\lambda$ . With the dashed green line is shown the model results with only infall with a constant  $\Lambda = 0.5$ .

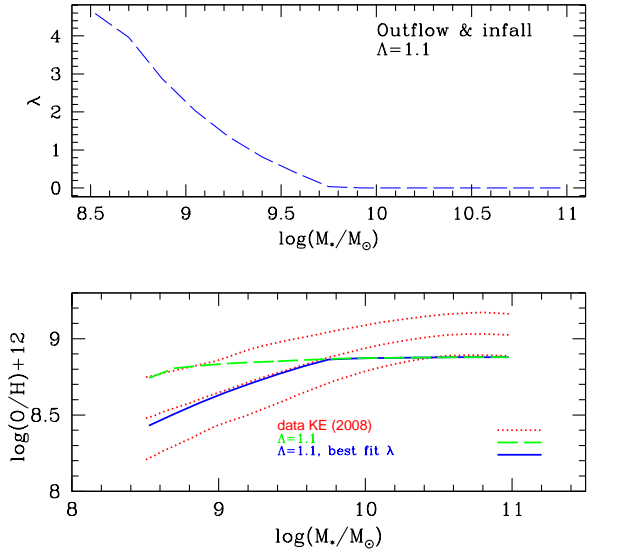
in galaxies. We note that for stellar masses smaller than  $10^{9.5}$ , we have  $\lambda \gg \Lambda$ . If we look at the MZ relation plot obtained with the best fits for  $\lambda$  and  $\Lambda$  as reported in the lower panel of Fig. 5, we see that for  $M_* < 10^{9.5} M_\odot$  the model with only infall does not reproduce at all the observed MZ relation. The reason for this is the fact that for values of  $\Lambda > 1$  the condition (10) holds, and this relation can be seen as a condition for  $\Lambda$  values:

$$\Lambda \leq \frac{1}{1 - \mu}, \quad (21)$$

In Fig. 6 the dashed area represents the forbidden  $\Lambda$  values, the rest of the area represents the allowed values. With the short dashed line we draw the values of  $\Lambda$  as a function of  $\log(M_*/M_\odot)$  relative to the predicted MZ relation of Fig. 5.

When  $\log(M_*/M_\odot)=8.5$  and  $\mu = 0.47$ , the  $\Lambda$  values must be  $< 1.88$ , whereas for  $\log(M_*/M_\odot)=11.0$  and  $\mu = 0.16$  we have that  $\Lambda < 1.2$ . Therefore, for the lower range of stellar masses considered, the derived  $\Lambda$  does not allow to reproduce the observed MZ relation. We conclude that the simple model solution concerning system with only infall varying with galactic stellar mass is unable to fit the MZ for small stellar mass galaxies even if we choose the best fit  $\Lambda$  values.

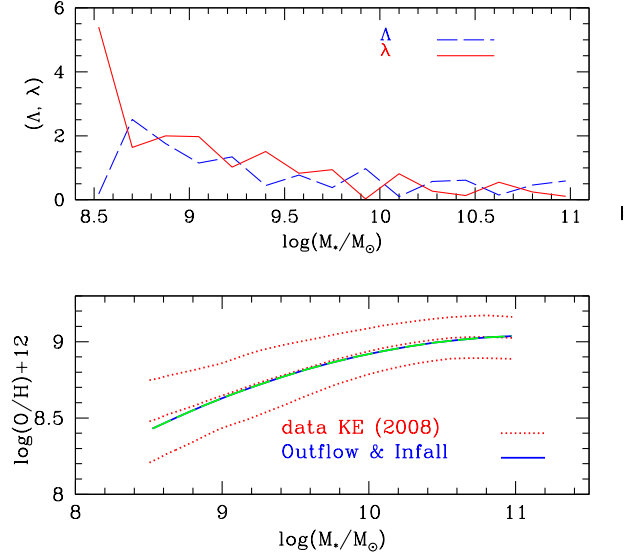
Concerning the solution of simple models with infall and outflow at the same time, we fix  $\Lambda = 0.5$  in the eq. (11), and we test which values of  $\lambda$  must be assumed to reproduce the MZ relation. As reported in the upper panel of Fig. 7, the best fit model for  $\lambda$  anti-correlates with the stellar mass  $M_*$ .



**Fig. 8.** *Upper panel:* The best  $\lambda$  values for a model with both infall and outflow.  $\lambda$  is given as a function of the galactic stellar mass  $M_*$  with fixed  $\Lambda=1.1$ . *Lower panel:* the best fit to the observed MZ relation and related standard deviation are indicated with the dotted red lines. With the blue solid line we show the best model results, obtained by considering both outflow in infall and by fixing  $\Lambda=1.1$  while varying  $\lambda$ . With the dashed green line is shown the model solution with only infall with  $\Lambda = 1.1$ .

In the lower panel we see that the MZ relation is well reproduced by the best fit model line (the blue solid one). In the MZ plot we also include the result of the model only with the infall parameter  $\Lambda$  equal to 0.5. This model solution is reported with the dashed line, and clearly it does not reproduce the MZ relation. Therefore, a variable infall efficiency or a fixed one coupled with a variable outflow as a function of the galactic stellar mass is required.

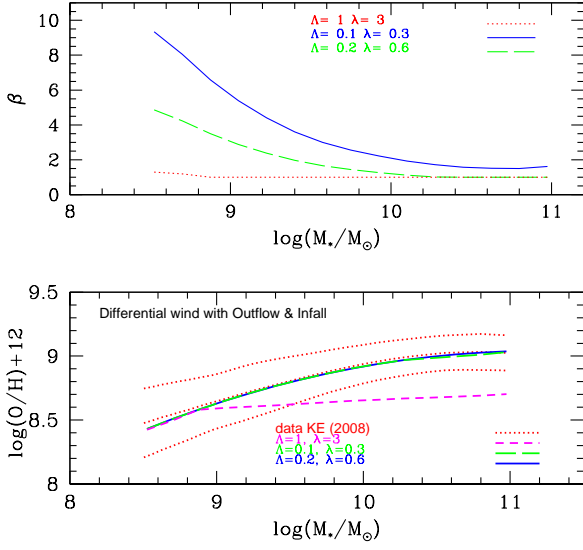
We analyze also the case with a constant infall  $\Lambda = 1.1$  and a variable outflow as reported in Fig. 8. In the upper panel we see the behavior of the model with the best fit  $\lambda$  values. The choice of this particular value of  $\Lambda$  is just arbitrary but it shows the effect of a variable galactic wind in presence of an infall with a fixed  $\Lambda$  value, large enough to make the metallicity of large galaxies to decrease. We see that for  $M_*$  larger than  $10^{9.8} M_\odot$ ,  $\lambda$  vanishes. The reason for this can be understood analyzing the lower panel of Fig. 8. As in Fig. 7 we also report the solution with only the infall. We note that the infall with  $\Lambda = 1.1$  is too strong and flattens the MZ relation for high stellar mass systems (the green dashed line). Therefore, if we couple this infall with an variable outflow, the best fit model (blue solid line) is able to fit the MZ relation for systems with small stellar mass values but certainly we cannot reproduce the part of the observed MZ where the infall was already too strong, namely for high galactic stellar masses. We find that the maximum value for the  $\Lambda$  parameter, when varying  $\lambda$  to reproduce the MZ relation, must be  $\leq 0.7$ .



**Fig. 9.** *Upper panel:* the best fit values for  $\Lambda$  and  $\lambda$  parameters in a model with infall and outflow, varying both the parameters at the same time. *Lower panel:* the bestfit to the observed MZ relation and related standard deviation are indicated with the dotted red lines. With the blue solid line we show the results of the best model adopting the  $\Lambda$  and  $\lambda$  values of the upper panel. With the dashed green line is shown the analytical fit of Maiolino et al. (2008).

The results obtained varying both the parameters are reported in Fig. 9. In the upper panel, the best fit parameters as a function of  $M_*$  are reported. These sets of best parameters lead to a difference in the MZ relation less than  $10^{-5}$  dex between the best fit of Maiolino et al. (2008) and simple model solutions with this choice. In fact, in the lower panel of Fig. 9 comparing the Maiolino's fit and our best fit, we see that the two lines overlap perfectly. It is worth noting that in most of the mass bins (at least for  $M_* < 10^{9.5} M_\odot$ ) is  $\lambda > \Lambda$ .

The last part of our work is focused on simple model solutions with infall and differential winds. We presented in Section 2, the analytical solution found by Recchi et al. (2008). Considering also in this case a model with primordial infall, i.e.  $Z_A = 0$  we study systems with 3 sets of parameters for  $\Lambda$  and  $\lambda$ :  $(\Lambda, \lambda) = (0.1, 0.3); (0.2, 0.6), (1, 3)$ . In the upper panel of Fig. 10 we show the variation of the possible  $\beta$  values which can be considered in order to reproduce the MZ relation. As expected, for all models the best fit for  $\beta$  anti-correlates with the galactic stellar mass, and the values of  $\beta$  are higher for the first model  $(\Lambda, \lambda) = (0.1, 0.3)$ . Both the first and the second model can reproduce the MZ relation. In the case with  $(\Lambda, \lambda) = (1, 3)$  instead, we are not able to reproduce the MZ relation, as shown in the lower panel of Fig. 10 with the magenta short dashed line. Moreover, as shown in Fig. 11 we fix the infall parameter at the value  $\Lambda = 0.1$  and we find that the upper limit value for  $\lambda$ , in order to fit the MZ relation, is  $\sim 0.6$  and in this case  $\beta$  ranges between 1 and 5.



**Fig. 10.** *Upper panel:* The best values for  $\beta$  in a model with both infall and outflow. The blue solid line is referred to a system with  $\Lambda = 0.1$  and  $\lambda = 0.3$ , the results for  $\Lambda = 0.2$  and  $\lambda = 0.6$  are indicated with the long dashed green line, and with the red short dashed line is shown the result with  $\Lambda = 1$  and  $\lambda = 3$ . *Lower panel:* the bestfit to the observed MZ relation and related standard deviation (the dotted red lines) are shown. With the blue solid line we show the results of the best model obtained by varying the ejection efficiency  $\beta$  and including also outflow and infall with  $(\Lambda, \lambda) = (0.2, 0.6)$ . The cases with  $(\Lambda, \lambda) = (0.1, 0.3)$  and  $(\Lambda, \lambda) = (1, 3)$  are indicated with the green long dashed line and the magenta short dashed line, respectively.

We also study the case in which  $\Lambda > \lambda$ , with this set of parameters:  $(\Lambda, \lambda) = (0.3, 0.1); (0.6, 0.2), (3, 1)$ . In this case, the solution is defined only for values of  $\mu$  such that:

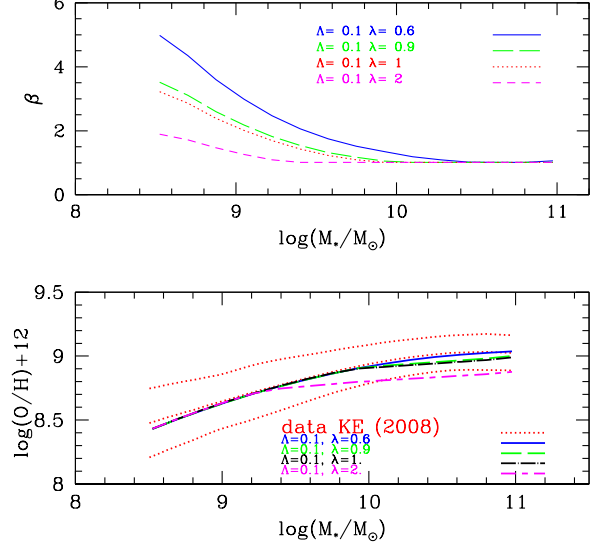
$$\mu > \frac{\Lambda - \lambda - 1}{\Lambda - \lambda}, \quad (22)$$

as in the case of the model with outflow and infall. Using the set  $(3, 1)$ ,  $\mu$  must be larger than 0.5. The observed average  $\mu$  spans the range between 0.16 and 0.47, then the  $(3, 1)$  set of parameters should be discarded. Both the first and the second model can reproduce the MZ relation, but the set  $(0.3, 0.1)$  requires values of  $\beta$  too high and unrealistic in comparison with the wind parameter  $\lambda$ , as shown in Fig. 12, and therefore it should also be rejected.

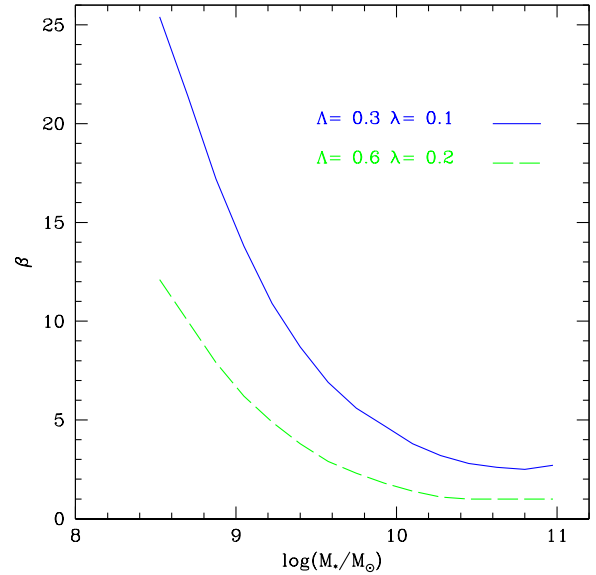
In Table 1 we summarize the permitted values of the parameters for reproducing the MZ relation for all the cases studied in this work.

## 5. Conclusions

In this paper we have explored different solutions for the MZ relation observed in SDSS galaxies. We adopted simple analytical models including infall, outflow and differential outflow besides the classical closed-box case. All the solutions of the



**Fig. 11.** *Upper panel:* The best values for  $\beta$  in a model with both infall and outflow. We fix here the value of  $\Lambda=0.1$  and vary  $\lambda$ . *Lower panel:* the bestfit to the observed MZ relation and related standard deviation are indicated with the dotted red lines. We fix the value of  $\Lambda=0.1$ . The other lines refer to the models of the upper panel.



**Fig. 12.** The best values for  $\beta$  in a model with both infall and outflow. The blue solid line is referred to a system with  $\Lambda = 0.1$  and  $\lambda = 0.3$ , whereas the results for  $\Lambda = 0.2$  and  $\lambda = 0.6$  are indicated with the long dashed green line.

analytical models contain the yield per stellar generation plus several parameters describing the different physical processes considered. We started by deriving the gas mass for the sample of the studied galaxies with known stellar galactic masses and star formation rates, by inverting the K98 law. Then, by means



**Table 1.** Permitted values of the parameters for reproducing the MZ relation.

Our Models	$\Lambda$	$\lambda$	$\beta$	$y_0$
Only outflow	/	$0.67 \leq \lambda \leq 5.55$	/	0.01
Only infall	see Fig. 6	/	/	0.01
Outflow + infall (varying both $\Lambda$ and $\lambda$ )	$0.10 \leq \Lambda \leq 2.51$	$0.11 \leq \lambda \leq 5.40$	/	0.01
Outflow + infall (fixed $\Lambda$ and varying $\lambda$ )	$0 < \Lambda \leq 0.70$	if $\Lambda = 0.70$ ; $0.16 \leq \lambda \leq 0.47$	/	0.01
		if $\Lambda = 0.01$ ; $0.66 \leq \lambda \leq 5.54$	/	0.01
Enriched Outflow + infall with $\Lambda < \lambda$ (fixed $\Lambda = 0.1$ )	0.1	$0.10 < \lambda \leq 0.60$	if $\lambda = 0.60$ ; $1.0 \leq \beta \leq 5.0$	0.01
			if $\lambda = 0.11$ ; $3.3 \leq \beta \leq 24.4$	0.01
Enriched Outflow + infall with $\Lambda > \lambda$ (fixed $\Lambda = 0.30$ and $\lambda = 0.10$ )	0.3	0.1	$2.5 \leq \beta \leq 25.4$	0.01
Enriched Outflow + infall with $\Lambda > \lambda$ (fixed $\Lambda = 0.6$ and $\lambda = 0.2$ )	0.6	0.2	$1.0 \leq \beta \leq 12.1$	0.01
Closed box+ variable IMF	/	/	/	$0.004 \leq y_0 \leq 0.007$

of these gas masses we derived the metallicity of each galaxy through the analytical models. At this point, we tried to vary the model parameters corresponding to the different physical processes to see whether the observed MZ could be reproduced. As it is well known, in order to obtain that metallicity increasing with galactic mass, the effective yield should decrease with decreasing mass. The effective yield can decrease because the outflow or infall become more important with decreasing mass or because the IMF becomes steeper. Some of these cases have been already studied by other authors but never all together, and here we add a new case with differential outflow with wind intensity increasing with decreasing mass. By differential outflow we mean a galactic wind where metals are lost preferentially. Then, we tested whether the required parameter variations were realistic in order to choose among the different solutions.

Our conclusions can be summarized as follows:

- The observed local MZ relation can be perfectly reproduced under the assumption of a closed-box and an effective yield varying from galaxy to galaxy. This means that either the nucleosynthesis is different or, more likely, that the IMF is different from galaxy to galaxy. The required yield variation is rather small and lies in the range 0.004-0.007, corresponding to a variation of the slope of the IMF (one-slope IMF defined in the mass range  $0.1-100M_{\odot}$ ) from  $\sim 1.35$  to  $1.5$  going from more to less massive galaxies. However, the variation of the IMF always implies variations in other physical quantities besides the O abundance and therefore, before accepting this solution, it would be necessary to test other observables such as colors, the color-magnitude diagrams and the M/L ratios, if available. Besides that, if we change the IMF we should in principle change also the conversion factor from SFR to  $\mu$  in the inversion of the Kennicutt law, which in our work has been calculated by adopting a Salpeter IMF in the mass range  $0.1-100 M_{\odot}$  (see Sect. 3).
- The observed MZ relation can be very well reproduced by a constant yield but a variable efficiency of the outflow, increasing from more to less massive galaxies. This is not a

new conclusion since it has been discussed before by several authors (Garnett 2000, Tremonti et al. 2004, Edmunds 2005). However, here we reached this conclusion by considering the most recent observational data. In particular, the wind parameter  $\lambda$ , which roughly represents the ratio between the wind rate and the SFR, should vary from 1 to 5.5, going from large to small galaxies, which is quite a reasonable range supported also by galactic wind observations (Martin et al. 2002). We have also found that outflows must play an important role especially for galaxies with stellar masses  $< 10^{10}M_{\odot}$ . This is consistent with other results on the efficiency of outflows as a function of galactic mass, which show that dwarf galaxies must have ejected a fraction of their present baryonic mass larger than massive galaxies (e.g. Gibson & Matteucci 1997; Calura et al. 2008).

- The local MZ relation cannot be reproduced by a variable infall efficiency no wind and constant IMF, so this solution should be rejected. This does not mean, however, that the infall is not important for the considered galaxies. Infall can be present but it is not the cause of the MZ relation. Moreover, an infall rate increasing with decreasing galactic mass it is not so easy to explain under physical basis.
- Models with variable infall and outflow rates can, in fact, well reproduce the observed local MZ relation. In this case, the outflow rate is generally larger than the infall rate.
- Differential galactic winds, where mostly metals are lost from the galaxy can also very well reproduce the observed MZ relation, provided that  $\lambda > \Lambda$ . In this case, the metal ejection efficiency  $\beta$  is always larger in smaller galaxies. On the other hand, when  $\Lambda > \lambda$  not all the values of  $\Lambda$  and  $\lambda$  are acceptable. Considering only the acceptable values for these parameters, we found that  $\beta$  values, able to reproduce the MZ relation, are generally higher than in the case with  $\Lambda < \lambda$ .

The assumption of a larger  $\beta$  in low-mass galaxies simply means that these galaxies are able to expel large fractions, relative to their total mass, of newly synthesized metals but only tiny fractions of pristine ISM, in agreement with

many hydrodynamical studies of star forming dwarf galaxies (e.g. MacLow & Ferrara 1999, D’Ercole & Brighenti 1999, Recchi et al. 2001).

- We did not explore the case in which the efficiency of star formation increases with galactic mass, since the SFR does not appear in the solutions of the analytical models. Moreover, such a solution has already been explored in detail by Calura et al. (2009), who showed that this assumption well reproduces the MZ relations both at low and high redshift. In addition, this variation of the star formation efficiency produces a downsizing effect in the star formation often invoked to explain the properties of ellipticals (Pipino & Matteucci, 2004).
- In conclusion, on the basis of this paper and of Calura et al.’s (2009) previous results, the most plausible solution is that the MZ relation is created by a variable star formation efficiency from galaxy to galaxy coupled to galactic winds (preferentially metal-enhanced) becoming more and more important in low mass galaxies. But as stressed by Edmunds (2005), explaining the mass-metallicity relation by a systematic increase in the star formation efficiency with galactic mass poses the problem of understanding the physical mechanism behind it. Several suggestions exist in the literature but the real physical origin of this correlation is not known. A possibility is that the higher the pressure in the ISM then the higher is the star formation efficiency, and the more massive galaxies have higher ISM pressure because of their deeper potential wells (see Elmegreen & Efremov 1997; Harfst et al. 2006).

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